SSLC CLASS NOTES

Chapter -5

PROBABILITY

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Random experiment

- The exact outcome cannot be predicted
- It has more than one possible outcome.
- It is not possible to predict the outcome in advance

Trial
- Performing Random Experiment
- Examples: (i) Tossing a coin(ii) Throwing a die

Outcome
- The result of a random experiment
- Examples: (i) Head (H), Tail (T)(ii) 1, 2, 3, 4, 5, 6

Sample space
- The set of all possible outcomes of a random experiment
- Examples: (i) S = {H, T}(ii) S = {1, 2, 3, 4, 5, 6}

Event
- Any possible outcome or combination of outcomes of a random experiment
- Examples: (i) A= {H } B = {T} (ii) A= {1,3,5} , B ={2,4,6} etc

Elementary event

Each outcome of the random experiment is called an elementary event

compound event

the events are obtained by combining two or more elementary events

Throwing a die

$E_1=\{1\}$, $E_2=\{5\}$ . .etc

Throwing a die

$E_1=\{1,3,5\}$,

$E_2=\{2,4,6\}$ . .etc

Favourable elementary events

- Throwing an unbiased die"getting an even number". .

2, 4, 6. are Favourable elementary eve

Probability

Total number of elementary events in the sample space S = **n(S)**

Number of elementary events favorable to event E = **n (E)**

Probability of event E = **P (E)**

$$P(E) = \frac{n(E)}{n(S)}$$

Sure event → **Surely occur in any trial of the experiment** → 1

Probability

Impossible event → **Will not occur on any account in any trial of the experiment** → 0

$$0 \leq P(A) \leq 1$$

ILLUSTRATIVE EXAMPLES

Example1: A die is rolled. Find the probability of getting.
(i) The number 5 (ii) an odd number (iii) a number greater than 2 (iv) a prime factor of 6

S = {rolling a die}
S = {1, 2, 3, 4, 5, 6}
n(S) = 6

Sol: (i) Let E be the event of getting the number 5
E = {5}, n (E) = 1
$P(E) = \frac{n(E)}{n(S)}$
$P(E) = \frac{1}{6}$

(ii) Let E be the event of getting an odd number
E = {1, 3, 5}, n (E) = 3
$P(E) = \frac{n(E)}{n(S)}$
$P(E) = \frac{3}{6}$

(iii) Let E be the event of getting a greater than 2
E = {3, 4, 5, 6}, n (E) = 4
$P(E) = \frac{n(E)}{n(S)}$
$P(E) = \frac{4}{6}$

(iv) Let E be the event of getting a prime factor of 6
E = {2, 3}, n (E) = 2
$P(E) = \frac{n(E)}{n(S)}$
$P(E) = \frac{2}{6}$

Example2:2: In tossing a fair coin twice, find the probability of getting.
(i) Two heads (ii) at least one head
(iii) No head (iv) exactly one tail

Sol: S = {tossing a fair coin twice}
S = {HH, HT, TH, TT}: n(S) = 4

(i) Getting two heads.
E = {HH} = n (E) = 1
$P(E) = \frac{n(E)}{n(S)}$
$P(E) = \frac{1}{4}$

(ii) Getting at least one head.
E = {HH, HT, TH} = n (E) = 3
$P(E) = \frac{n(E)}{n(S)}$
$P(E) = \frac{3}{4}$

(iii) Getting no head
E = {TT} = n (E) = 1

P (E) = $\frac{n(E)}{n(S)}$

P (E) = $\frac{1}{4}$

(iv) Getting exactly one head.

E = {HT, TH} = n (E) = 2

P (E) = $\frac{n(E)}{n(S)}$

P (E) = $\frac{2}{4}$

Example3: In tossing three fair coins together what is the probability of getting

(i) All tails (ii) at least one tail

(iii) At most one tail (iv) at most two heads.

Sol: S = {tossing three fair coins together}

S = {HHH, HHT, THH, THT, TTH, THT, HTT, TTT}

n(S) = 8

(i) Getting all tails.

E = {HHH} = n (E) = 1

P (E) = $\frac{n(E)}{n(S)}$

P (E) = $\frac{1}{8}$

(ii) Getting at least one tail.

E = {HHH, HHT, HTH, THH, HTT, THT, TTH} ⇒ n (E) = 1

P (E) = $\frac{n(E)}{n(S)}$

P (E) = $\frac{7}{8}$

(iii) Getting at most one tail.

E = {HHH, HHT, HTH, THH} ⇒ n (E) = 4

P (E) = $\frac{n(E)}{n(S)}$

P (E) = $\frac{4}{8}$

(iv) Getting at most two heads.

E = {HHT, HTH, THH, HTT, THT, TTH, TTT} - n (E) = 7

P (E) = $\frac{n(E)}{n(S)}$

P (E) = $\frac{7}{8}$

Example4: Two unbiased dice are rolled once. What is the probability of getting?

(i) A doublet (ii) a sum equal to 7 (iii) a sum less than 10?

Sol: S = {Rolling two unbiased dice}

S = { (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) (3,1) (3,2) (3,3) (3,4) (3,5) (3,6) (4,1) (4,2) (4,3) (4,4) (4,5) (4,6) (5,1) (5,2) (5,3)(5,4) (5,5) (5,6) (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)}

n(S) = 36

(i) Getting a doublet.

E = {(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)}, n(E) = 6

P (E) = $\frac{n(E)}{n(S)}$

P (E) = $\frac{6}{36}$

(ii) Getting a sum equal to 7

E = {(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)}, n(E) = 6

P (E) = $\frac{n(E)}{n(S)}$
P (E) = $\frac{6}{36}$
(iii)Getting a sum less than 10
E = { (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) (3,1) (3,2) (3,3) (3,4) (3,5) (3,6) (4,1) (4,2) (4,3) (4,4) (4,5) (5,1) (5,2) (5,3)(5,4) (6,1) (6,2) (6,3)}
n (E) = 30
P (E) = $\frac{n(E)}{n(S)}$
P (E) = $\frac{30}{36}$

Example5: There are 30 boys and 25 girls in a class. A student is chosen at random. Find the probability that the chosen student is a (i) boy (ii) girl

Sol: S = {Total number of students} - n(S) = 55
(i) Choosing a boy
n (E) = 30
P (E) = $\frac{n(E)}{n(S)}$
P (E) = $\frac{30}{55}$
(ii) Choosing a girl
n (E) = 25
P (E) = $\frac{n(E)}{n(S)}$
P (E) = $\frac{25}{55}$

Example6: What is the probability that a leap year selected will contain 53 Sundays?

Sol: Number of days in a leap year = 366 days
⇒366 days = 52 weeks and 2 days
The remaining 2 days can be
(i) Sunday and Monday (ii) Monday and Tuesday
(iii) Tuesday and Wednesday (iv) Wednesday and Thursday
(v) Thursday and Friday (vi) Friday and Saturday
(vii) Saturday and Sunday.
n(S) = 7
n (E) = 2
P (E) = $\frac{n(E)}{n(S)}$
P (E) = $\frac{2}{7}$

Example7: A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball is twice that of drawing a red ball, then find the number of blue balls in the bag.

Number of red ballad = 6
Let the number of blue balls = x
Total number of balls n(S) = 6+x
P (drawing a red ball) = $\frac{6}{6+x}$
P (drawing a blue ball) = $\frac{x}{6+x}$
$\frac{x}{6+x} = 2\left[\frac{6}{6+x}\right]$
$\frac{x}{6+x} = \frac{12}{6+x}$
$x(6+x) = 12(6+x)$
$x(6+x) = 12(6+x)$

$6x + x2 = 72 + 12x$
$x^2 – 6x - 72 = 0$
$x^2 – 12x + 6x - 72 = 0$
$x(x – 12) + 6(x - 12) = 0$
$(x – 12)(x + 6)= 0$
$x = 12$ or $x = -6$
∴ The number of blue balls = 12

Example8: From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting
(i) A queen (ii) a red king (iii) a diamond (iv) a heart.

Sol: $n(S) = 52$

(i) A queen $n(E) = 4$
$P(E) = \frac{n(E)}{n(S)}$
$P(E) = \frac{4}{52}$
(ii) A red king, $n(E) = 2$
$P(E) = \frac{n(E)}{n(S)}$
$P(E) = \frac{2}{52}$
(iii) A diamond, $n(E) = 13$
$P(E) = \frac{n(E)}{n(S)}$
$P(E) = \frac{13}{52}$
(iv) A heart
$n(E) = 13$
$P(E) = \frac{n(E)}{n(S)}$
$P(E) = \frac{13}{52}$

Note: the sum of the probabilities of all the elementary events of an experiment is 1.
Example1: Tossing a coin
Elementary events are: $E_1 = \{H\}$, $E_2 = \{T\}$
$P(E_1) + P(E_2) = \frac{1}{2} + \frac{1}{2} = 1$
Example2: Throwing a die
Elementary events are: $E_1=\{1\}, E_2=\{2), E_3=\{3\}, E_4 =\{4\}, E_5 =\{5\}, E_6 = \{6\}$
$P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$

Exercise 5.1

1. A die is rolled. Find the probability of getting
   (i) The number 4 (ii) a square number
   (iii) A cube number (iv) a number greater than 1

$S = \{1, 2, 3, 4, 5, 6\}$ - $n(S) = 6$
(i). $E = \{\text{the number } 4\} = \{4\}$
$n(E) = 1$
$P(E) = \frac{n(E)}{n(S)} \Rightarrow \mathbf{P(E) = \frac{1}{6}}$
(ii). $E = \{\text{a square number}\} = \{1, 4\}$
$n(E) = 2$
$P(E) = \frac{n(E)}{n(S)} \Rightarrow \mathbf{P(E) = \frac{2}{6}}$

(ii). E = {a cube number} = {1}
n (E) = 1
P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{1}{6}}$
(iv). E = {a number greater than 1} = {2, 3, 4, 5, 6}
n (E) = 5
P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{5}{6}}$

2. Two coins are tossed together. What is the probability of getting?
(i) No tail (ii) at most two tails
(iii) Exactly one head (iv) at least one tail
S = {HH, HT, TH, TT} n(S) = 4
(i). E = {no tail} = {HH} - n (E) = 1
P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{1}{4}}$
(ii). E = { at most two tails }={HH,TH,HT,TT} - n(E) = 4
P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{4}{4} = 1}$
(iii). E = {exactly one head} = {TH, HT} - n (E) = 2
P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{2}{4}}$
(iv). E = {at least one tail}={TH,HT,TT} - n(E) = 3
P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{3}{4}}$

3. Three coins are tossed together. Find the probability of getting.
(i) At least one head (ii) at most two heads
(iii) No head (iv) all heads
S = {HHH, HHT, THH, HTH, TTH, THT, HTT, TTT} - n(S) = 8
(i). E = { at least one head }={HHH,HHT,HTH,THH,TTH,THT,HTT} - n(E) = 7
P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{7}{8}}$
(ii). E = { at most two heads }={HHT,HTH,THH,TTH,THT,HTT,TTT} - n(E) = 7
P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{7}{8}}$
(iii). E = {no head} = {TTT} - n (E) = 1
P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{1}{8}}$
(iii). E = {all heads} = {HHH} - n (E) = 1
P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{1}{8}}$

4. Two dice are thrown together. Find the probability of getting.
(i) A sum equal to 8 (ii) a sum less than 12
(iii) The sum divisible by 4 (iv) a product of 12
(v) A product less than 20 (vi) the product divisible by 5
S = { (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) (3,1) (3,2) (3,3) (3,4) (3,5) (3,6) (4,1) (4,2) (4,3) (4,4) (4,5) (4,6) (5,1) (5,2) (5,3) (5,4) (5,5) (5,6) (6,1) (6,2) (6,3) (6,4) (6,5) (6,6) } - n(S) = 36
(i). E = { a sum equal to 8}={(2,6) (3,5) (4,4) (5,3) (6,2)} - n(E) = 5
P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{5}{36}}$
(ii). E = {a sum less than 12}
={(1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) (3,1) (3,2) (3,3) (3,4) (3,5) (3,6) (4,1) (4,2) (4,3)(4,4) (4,5) (4,6) (5,1) (5,2)(5,3) (5,4) (5,5) (5,6) (6,1) (6,2) (6,3) (6,4) (6,5)} - n(E) = 35

P (E) = $\frac{n(E)}{n(S)}$ - **P (E) =** $\mathbf{\frac{35}{36}}$

(iii). E ={ the sum divisible by 4}={(1,3) (2,2) (2,6) (3,1) (3,5) (4,4)} (5,3) (6,2) (6,6) }

n (E) = 9

P (E) = $\frac{n(E)}{n(S)} \Rightarrow$ **P (E) =** $\mathbf{\frac{9}{36}}$

(iv). E ={ a product of 12 } ={(2,6) (3,4) (4,3) (6,2)} - n(E) = 4

P (E) = $\frac{n(E)}{n(S)} \Rightarrow$ **P (E) =** $\mathbf{\frac{4}{36}}$

(v). E ={ a product less than 20}={(1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) (3,1) (3,2)(3,3)(3,4)(3,5)(3,6) (4,1) (4,2) (4,3) (4,4) (5,1) (5,2) (5,3) (6,1) (6,2) (6,3)}

n (E) = 28

P (E) = $\frac{n(E)}{n(S)} \Rightarrow$ **P (E) =** $\mathbf{\frac{28}{36}}$

(vi). E ={ the product divisible by 5}={ (1,5) (2,5)(3,5)(4,5) (5,1) (5,2) (5,3) (5,4) (5,5) (5,6)(6,5)} - n(E) = 11

P (E) = $\frac{n(E)}{n(S)} \Rightarrow$ **P (E) =** $\mathbf{\frac{11}{36}}$

5. A number is selected at random from 1 to 50. What is the probability that it is?
   (i) A prime number (ii) n o t a perfect cube
   (iii) A perfect square (iv) a triangular number
   (v) A multiple of 6 (vi) not a multiple of 2

S – {A number is selected at random from 1 to 50} - n(S) = 50

(i). E ={ a prime number } ={ 2,3,5,7,11,13,17,19, 23,29,31, 37, 41, 43,47} - n(E) = 15

P (E) = $\frac{n(E)}{n(S)} \Rightarrow$ **P (E) =** $\mathbf{\frac{15}{50}}$

(ii). E = {not a perfect cube}
= {2,3,4,5,6,7,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23, 24,25,26,28,29,30,31,32,33, 34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50} - n(E) = 47

P (E) = $\frac{n(E)}{n(S)} \Rightarrow$ **P (E) =** $\mathbf{\frac{47}{50}}$

(iii). E = {A perfect square} = {1, 4, 9, 16, 25, 36, 49} - n (E) = 7

P (E) = $\frac{n(E)}{n(S)} \Rightarrow$ **P (E) =** $\mathbf{\frac{7}{50}}$

(iv). E = {a triangular number} = {1, 3, 6, 10, 15, 21, 28, 36, 45} - n (E) = 9

P (E) = $\frac{n(E)}{n(S)}$ = **P (E) =** $\mathbf{\frac{9}{50}}$

(v). E = {A multiple of 6} = {6, 12, 18, 24, 30, 36, 42, 48} **-** n (E) = 8

P (E) = $\frac{n(E)}{n(S)}$ = **P (E) =** $\mathbf{\frac{8}{50}}$

(vi). E ={ not a multiple of 2} = {2,4,6,8,10,12,14,16,18,20,22,24,26,28, 30, 32,34,36,38,40,42,44,46,48,50} = n(E) = 25

P (E) = $\frac{n(E)}{n(S)}$ = **P (E) =** $\mathbf{\frac{25}{50}}$

6. One card is drawn randomly from a well shuffled pack of 52 playing cards. Find the probability that the drawn card is
   (I) a red colored card (ii) not a black colored card
   (iii) Not a diamond (iv) not an ace
   (v) A black king (vi) a heart with number less than 10.

S – {One card is drawn randomly from a well shuffled pack of 52 playing cards} - n(S) = 52

(i). E = {a red colored card} - n (E) = 26

P (E) = $\frac{n(E)}{n(S)}$ = **P (E) =** $\mathbf{\frac{26}{52}}$

(ii). E = {not a black colored card} - n (E) = 26

P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{26}{52}}$

(iii). E = {not a diamond} = n (E) = 39

P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{39}{52}}$

(iv). E = {not an ace} = n (E) = 48

P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{48}{52}}$

(v). E = {a black king} = n (E) = 2

P (E) = $\frac{n(E)}{n(S)}$ = **P (E) =** $\mathbf{\frac{2}{52}}$

(vi). E = {a heart with number less than 10} = n (E) = 8

P (E) = $\frac{n(E)}{n(S)}$ ⇒**P (E) =** $\mathbf{\frac{8}{52}}$

7. A two-digit number is formed with the digits 2, 5 and 7, where repetition of digits is not allowed. Find the probability that the number so formed is
   (i) A square number (ii) divisible by 3
   (iii) Greater than 52 (iv) less than 57
   (v) Less than 25 (vi) a whole number

   S – {A two-digit number is formed with the digits 2, 5 and 7, without repetition}

   n(S) = $2\mathbf{P_1}$x$3P_1$ ⇒n(S) = 2x3 = 6 [(25, 27, 52, 57, 72, 75}

T	U
$2\boldsymbol{P_1}$	$3P_1$
2	3

   (i). E = {a square number} ⇒n (E) = 1

   P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{1}{6}}$

   (ii). E = {divisible by 3} ⇒n (E) = 4

   P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{4}{6}}$

   (iii). E = {greater than 52} ⇒ n (E) = 3

   P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{3}{6}}$

   (iv). E ={less than 57} ⇒ n (E) = 3

   P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{3}{6}}$

   (iv). E ={less than 25} ⇒ n (E) = 0

   P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) = 0**

   (iv). E = {a whole number} ⇒ n (E) = 6

   P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{6}{6}}$ **= 1**

8. Nine rotten mangoes are mixed with 30 good ones. One mango is chosen at random. What is the probability of choosing a
   (i) Good mango (ii) rotten mango

   S – {One mango is chosen at random out of 39} ⇒ n(S) = 39

   (i). E = {A good mango is chosen} ⇒ n (E) = 30

   P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{30}{39}}$

   (I). E = {rotten mango} ⇒ n (E) = 9

   P (E) = $\frac{n(E)}{n(S)}$ ⇒ **P (E) =** $\mathbf{\frac{9}{39}}$

9. During holi festival, Sonali filled 7 bottles with different colored water - red, blue, green, pink, yellow, purple and orange.
   One bottle is selected at random. What is the probability of choosing the bottle with?
   (i) Orange colour (ii) not yellow colour (iii) brown colour

(iv) Either red or green (v) neither yellow nor pink

S – {One bottle is selected at random out of 7} ⇒ n(S) = 9

(i). E = {orange colour} ⇒ n (E) = 1

$P(E) = \frac{n(E)}{n(S)} \Rightarrow \mathbf{P(E) = \frac{1}{7}}$

(ii). E = {not yellow colour} ⇒ n (E) = 6

$P(E) = \frac{n(E)}{n(S)} \Rightarrow \mathbf{P(E) = \frac{6}{7}}$

(iii). E = {brown colour} ⇒ n (E) = 0

$P(E) = \frac{n(E)}{n(S)} \Rightarrow \mathbf{P(E) = 0}$

(iv). E = {either red or green} ⇒ n (E) = 2

$P(E) = \frac{n(E)}{n(S)} \Rightarrow \mathbf{P(E) = \frac{2}{7}}$

(iv). E = {neither yellow nor pink} ⇒ n (E) = 5

$P(E) = \frac{n(E)}{n(S)} \Rightarrow \mathbf{P(E) = \frac{5}{7}}$

10. A box contains 144 pens of which 20 are defective and the others are good. A person will buy a pen if it is good and will not buy if it is defective. The shopkeeper draws one pen from the box at random and gives it to the person. What is the probability that the person.
(i) Will buy it (ii) will not buy it?

S – {Draws one pen from the box containing 144 pens} ⇒n(S) = 144

(i). E = {will buy it} ⇒ n (E) = 124

$P(E) = \frac{n(E)}{n(S)} \Rightarrow \mathbf{P(E) = \frac{124}{144}}$

(I). E = {will not buy it} ⇒ n (E) = 20

$P(E) = \frac{n(E)}{n(S)} \Rightarrow \mathbf{P(E) = \frac{20}{144}}$

Complementary events

The event occurs only when the other not occurs and vice versa is called complimentory events	Example: Throwing a die E_1 -{ Geting even number} E_2 -{Getting odd number} The complimentory event of E is denoted by $\bar{E}$	$\mathbf{P(\bar{E}) = 1 - P(E)}$

Mutually exclusive events

The occurance of one event prevents or excludes the occurance of other event	Example: Throwing a die E_1 -{ Getting number less than 3} E_2 -{Getting number more than 4} E_1 and $E_2$are mutually exclusive events	$\mathbf{E_1 \cap E_2 = \emptyset}$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

Example Questions

Example1: If the probability of winning a game is 0.3, what is the probability of losing it?

Sol: Probability of winning a game $P(A) = 0.3$

Probability of losing: $p(A^1) = ?$

$P(A) = 1 - P(A^1)$

$0.3 = 1 - P(A^1)$

$P(A^1) = 1 - 0.3$

$P(A^1) = 0.7$

Example2: If A is an event of a random experiment such that P(A) : P(A) = 5:11, then find (i) P(A) and P(A), (ii) Verify that(A) + P(A) = 1

Sol: (i) $P(A): P(A^1) = 5:11$

$\Rightarrow \frac{P(A)}{P(A^1)} = \frac{5}{11}$

$\Rightarrow 11P(A) = 5P(A^1)$

$\Rightarrow 11P(A) = 5[1 - P(A)]$ [$\because P(A) + P(A^1) = 1$]

$\Rightarrow 11P(A) = 5 - 5P(A)$

$\Rightarrow 16P(A) = 5$

$\Rightarrow P(A) = \frac{5}{16}$

$\Rightarrow P(A^1) = 1 - \frac{5}{16}$

$\Rightarrow P(A^1) = \frac{11}{16}$

(ii) $P(A) + P(A^1) = \frac{5}{16} + \frac{11}{16} = \frac{5+11}{16} = \frac{16}{16} = 1$

ILLUSTRATIVE EXAMPLES

Example1: Two coins are tossed simultaneously. Find the probability that either exactly two heads or at least one tail turn.

Sol: $S = \{HH, HT, TH, TT\}$, $n(S) = 4$

Let, A be the event of getting exactly two heads = $\{HH\} \Rightarrow n(A) = 1$

$P(A) = \frac{n(A)}{n(S)} \Rightarrow P(A) = \frac{1}{4}$

Let, B be the event of getting at least one tail. $\Rightarrow B = \{HT, TH, TT\} \Rightarrow n(B) = 3$

$P(B) = \frac{n(B)}{n(S)} \Rightarrow P(B) = \frac{3}{4}$

A and B are mutually exclusive events

$P(AUB) = p(A) + P(B) = \frac{1}{4} + \frac{3}{4} = 1$

Example 2: Two dice are thrown together. Find the probability that a sum greater than 10 or a sum less than 5 turn up.

Sol: $S = \{(a, b)/a, b=1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 36$

Let, A be the event of getting a sum greater than 10 $\Rightarrow A = \{(5, 6)(6, 5)(6, 6)\} \Rightarrow n(A) = 3$

$P(A) = \frac{n(A)}{n(S)} \Rightarrow P(A) = \frac{3}{36}$

Let, B be the event of getting a sum less than 5

$\Rightarrow B = \{(1,1)(1,2)(1,3)(2,1)(2,2)(3,1)\} \Rightarrow n(B) = 6$

$P(B) = \frac{n(B)}{n(S)} \Rightarrow P(B) = \frac{6}{36}$

A and B are mutually exclusive events

∴P (AUB) = P (A) + P (B)

∴P (AUB) = $\frac{3}{36}+\frac{6}{36}$

∴P (AUB) = $\frac{9}{36}$

Example 3: A card is drawn randomly from a well shuffled pack of 52 cards. Find the probability of getting a red card or a black king or a club with an even number.

Sol: S = {A card is drawn randomly from a well shuffled pack of 52 cards} ⇒ n(S) = 52

Let A be the event of getting a red card ⇒ n (A) = 26

P (A) = $\frac{n(A)}{n(S)}$ ⇒ P (A) = $\frac{26}{52}$

Let B be the event of getting a black king ⇒ n (B) = 2

P (B) = $\frac{n(B)}{n(S)}$ ⇒ P (B) = $\frac{2}{52}$

Let C be the event of getting a club with an even number ⇒ C = {2, 4, 6, 8, 10} ⇒ n(C) = 5

P(C) = $\frac{n(C)}{n(S)}$ ⇒ P(C) = $\frac{5}{52}$

A, B, C are mutually exclusive events

∴ P (AUBUC) = P (A) + P (B) + P(C)

⇒ P (AUBUC) = $\frac{26}{52}+\frac{2}{52}+\frac{5}{52}=\frac{33}{52}$

Example 4: A letter is chosen at random from the letters of the word MATHEMATICIAN. Find the probability that the chosen letter is M or A.

Sol: There are 13 letters in the word MATHEMATICIAN. ∴ n(S) = 13

Let A be the event of getting the letter M ⇒ n (A) = 2

P (A) = $\frac{n(A)}{n(S)}$ ⇒ P (A) = $\frac{2}{13}$

Let B be the event of getting the letter A ⇒ n (B) = 3

P (B) = $\frac{n(B)}{n(S)}$ ⇒ P (B) = $\frac{3}{13}$

A and B are mutually exclusive events

∴P (AUB) = P (A) + P (B)

⇒ P (AUB) = $\frac{2}{13}+\frac{3}{13}=\frac{5}{13}$

Example 5: In a class, 40% of the students are members of the eco club and 25% of the students are members of the mathematics club. If a student is selected at random from the class, what is the probability that the student is not a member of either of the clubs?

Sol: Let n(s) = 100 and A be the event of selecting a member of eco club

N(S) = 100; n (A) = 40

P (A) = $\frac{n(A)}{n(S)}$ ⇒ P (A) = $\frac{40}{100}$

Let B be the event of selecting a member of mathematics club ⇒ n (B) = 25

P (B) = $\frac{n(B)}{n(S)}$ ⇒ P (B) = $\frac{25}{100}$

Since, A and B are mutually exclusive events

The probability of selecting a student who is a member of eco club or mathematics club is

P (AUB) = P (A) + P (B) = $\frac{65}{100}$

∴ The probability of selecting a student who is not a member of either of the clubs:

1 – P (AUB) = 1 - $\frac{65}{100}=\frac{35}{100}$

Example6: A random experiment has only three pairs of mutually exclusive events among A, B and C. If P (A) = $\frac{3}{2}$ P (B) and P(C) = $\frac{1}{2}$ P (A) then find P (B).

Sol: Let P (B) be x

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$\Rightarrow$P (A) = $\frac{3}{2}$P (B) = $\frac{3}{2}x$
P(C) = $\frac{1}{2}$P (A) = $\frac{1}{2}\left(\frac{3}{2}x\right) = \frac{3}{4}x$
A, B, C are mutually exclusive events
And P (A) + P (B) + P(C) = 1 $\Rightarrow \frac{3}{2}x + x + \frac{3}{4}x = 1$
$\frac{13x}{4} = 1$
13x = 4
x = $\frac{4}{13}$ $\Rightarrow$ P (B) = $\frac{4}{13}$

Exercise 5.2

1. The probability that it will rain on a particular day is 0.64 what is the probability that it will not rain on that day?
Sol: probability that it will rain P (E) = 0.64
Probability that it will not rain P ($\overline{E}$) = 1- 0.64 = 0.36
2. The probability of picking a non-defective item from a sample is $\frac{7}{12}$. Find the probability of picking a defective one.
Sol: probability of picking a non-defective item P (E) = $\frac{7}{12}$
Probability of picking a defective one P ($\overline{E}$) = $1 - \frac{7}{12} = \frac{5}{12}$
3. If A is an event of a random experiment such that P (A): P (A) = 6:15, then find (I) P (A) (ii) P ($\overline{A}$).
Sol: P (A): P ($\overline{A}$) = 6:15
$\Rightarrow \frac{P(A)}{P(\overline{A})} = \frac{6}{15}$
$\Rightarrow 15P(A) = 6P(\overline{A})$
$\Rightarrow 15P(A) = 6[1 - P(A)]$
$\Rightarrow 15P(A) = 6 - 6P(A)]$
$\Rightarrow 21P(A) = 6$
$\Rightarrow$ (I) $P(A) = \frac{6}{21}$
$\therefore$ (ii)$P(\overline{A}) = 1 - \frac{6}{21}$
$\therefore$ $P(\overline{A}) = \frac{15}{21}$
4. If A and B are the mutually exclusive events such that P (A) = $\frac{3}{5}$ and P (B) = $\frac{2}{7}$ find P (AUB).
Sol: P (AUB) = P (A) + P (B)
P (AUB) = $\frac{3}{5} + \frac{2}{7} = \frac{21+10}{35} = \frac{31}{35}$
5. Two coins are tossed simultaneously. Find the probability that either both heads and both tails occur.
Sol: S – {Two coins are tossed simultaneously} ⇒ S – {HH, HT, TH, TT} ⇒ n(S) = 4
E – {Getting either both heads or both tails} ⇒ E – {HH, TT} ⇒ n (E) = 2
P (E) = $\frac{n(E)}{n(S)}$ ⇒ P (E) = $\frac{2}{4}$
6. When a die is thrown, find the probability that either an odd number or a square number occurs.
S – {Throwing a die} ⇒ S – {1, 2, 3, 4, 5, 6} ⇒ n(S) = 6
E – {Getting either an odd number or a square number} ⇒ E – {1, 3, 4, 5} ⇒ n (E) = 4
P (E) = $\frac{n(E)}{n(S)}$ ⇒ P (E) = $\frac{4}{6}$

7. One number card is chosen randomly from the number cards 1 to 25. Find the probability that it is divisible by 3 or 11.
Sol: S – {One number card is chosen randomly from the number cards 1 to 25}
$\Rightarrow$ n(S) = 25
E – {The even that getting the number 3 is divisible by 3 or}
E – { 3,6,9,11,12,15,18,21,22,24} $\Rightarrow$ n(E) = 10
P (E) = $\frac{n(E)}{n(S)} \Rightarrow$ P (E) = $\frac{10}{25}$

8. Two dice are thrown simultaneously. Find the probability that the sum of the numbers on the faces is neither divisible by 4 nor by 5.
S – {Two dice are thrown simultaneously} $\Rightarrow$ S = {(a, b)/ a, b =1, 2, 3, 4, 5, 6} $\Rightarrow$ n(S) = 36
E – {the sum of the numbers on the faces is neither divisible by 4 nor by 5}
E={(1,1)(1,2)(1,5)(1,6)(2,1)(2,4)(2,5)(3,3)(3,4)(3,6)(4,2)(4,3)(4,5)(5,1)(5,2)(5,4)(5,6)
(6, 1)(6, 3)(6, 5)} $\Rightarrow$ n (E) = 20
P (E) = $\frac{n(E)}{n(S)} \Rightarrow$ P (E) = $\frac{20}{36}$

ILLUSTRATIVE EXAMPLES

Example1: Cards are numbered 1 to 25. Two cards are drawn one after the other. Find the probability that the number on one card is a multiple of 7 and on the other is a multiple of 11.
Sol: S = {Two cards are drawn from a cards are numbered 1 to 25}
n(S) = $^{25}C_2 = \frac{25x24}{2}$ = 300
A = {one card is a multiple of 7 and on the other is a multiple of 11}
n (A) = $^{3}C_1$ x $^{2}C_1$ = 3x2 = 6
P (A) = $\frac{n(A)}{n(S)}$ $\Rightarrow$ P (A) = $\frac{6}{300}$

Example2: Four cards are drawn at random from a well shuffled pack of 52 cards.
i) Find the probability that they all are diamonds.
ii) Find the probability that there are two spades and two hearts.
Sol: S = {Four cards are drawn at random from a well shuffled pack of 52 cards}
n(S) = $\frac{52x51x50x49}{4x3x2x1}$
A = {all are diamonds}
n (A) = $^{13}C_4 = \frac{13x12x11x10}{4x3x2x1}$
P (A) = $\frac{n(A)}{n(S)}$
P (A) = $\frac{\frac{13x12x11x10}{4x3x2x1}}{\frac{52x51x50x49}{4x3x2x1}} = \frac{11}{4165}$
B = {two spades and two hearts}
n (B) = $^{13}C_2$ x $^{13}C_2 = \frac{13x12}{2x1}$ x $\frac{13x12}{2x1}$
P (B) = $\frac{n(B)}{n(S)}$
P (B) = $\frac{\frac{13x12}{2x1} x \frac{13x12}{2x1}}{\frac{52x51x50x49}{4x3x2x1}} = \frac{468}{20825}$

Example3: A box has 4 red, and 3 black marbles. Four marbles are picked up randomly. Find the probability that,
(a) A = two marbles are red (b) B = all the marbles are red
(c) C = all the marbles are black.
Sol: S = {Four marbles are picked up from a box containing 4 red, and 3 black marbles}

$n(S) = {}^7C_4 = \frac{7x6x5x4}{4x3x2x1} = 35$

(i) A = {two marbles are red}

$n(A) = {}^4C_2 \times {}^3C_2 = \frac{4x3}{2} \times \frac{3x2}{2} = 18$

$P(A) = \frac{n(A)}{n(S)} \Rightarrow P(A) = \frac{18}{35}$

(ii) B = {all the marbles are red}

$n(B) = {}^4C_4 = 1$

$P(B) = \frac{n(B)}{n(S)} \Rightarrow P(B) = \frac{1}{35}$

(iii) C = {all the marbles are black}

C is an impossible event.

$\therefore P(C) = 0$

Example4: A box has 6 red, 7 white and 7 black balls. Two balls are drawn at random. Find the probability that the balls are red or both the balls are black.

Sol: S = {Two balls are drawn from a box containing 6 red, 7 white and 7 black balls}

$n(S) = {}^{20}C_2 = \frac{20x19}{2} = 190$

A = {the balls are red} $\Rightarrow n(A) = {}^6C_2 = \frac{6x5}{2} = 15$

$P(A) = \frac{n(A)}{n(S)} \Rightarrow P(A) = \frac{15}{190}$

B = {the balls are black} $\Rightarrow n(B) = {}^7C_2 = \frac{7x6}{2} = 21$

$P(B) = \frac{n(B)}{n(S)} \Rightarrow P(B) = \frac{21}{190}$

A and B are mutually exclusive

$\therefore P(AUB) = P(A) + P(B) = \frac{15}{190} + \frac{21}{190} = \frac{36}{190}$

Example5: India, Nepal, Pakistan, Bhutan and Sri Lanka have competed in volleyball tournament. The winners get in order of merit I, II or III prize. What is the probability of India securing any one of the prizes?

Sol: $n(S) = {}^5P_3 = 5x4x3 = 60$

A = {India securing anyone of the prizes}

$n(A) = 3x\,{}^4P_2 \Rightarrow n(A) = 36$ [$1x^4P_2 + 1x^4P_2 + 1x^4P_2$]

$P(A) = \frac{n(A)}{n(S)} \Rightarrow P(A) = \frac{36}{60}$

Exercise 5.3

1. There are 2 red and 2 yellow flowers in a basket. A child picks up at random three flowers. What is the probability of picking up both the yellow flowers?

S – {Picking up 3 flowers from a basket containing 2 red and 2 yellow flowers}

$n(S) = 4C_3$

$nC_r = \frac{n!}{(n-r)!r!}$

${}^4C_3 = \frac{4!}{(4-3)!3!}$

${}^4C_3 = \frac{4!}{1!3!} = 4$

$n(S) = 4$

E – {Picking up both the yellow flowers}

$n(E) = {}^2C_1 \times {}^2C_2$

$n(E) = 2 \times 1$

$n(E) = 2$

$P(E) = \frac{n(E)}{n(S)} \Rightarrow P(E) = \frac{2}{4}$

R	Y
$2C_1$	$2C_2$
2	1

2. Shear is one member of a group of 5 persons. If 3 out of these 5 persons is to be chosen for a committee, find the probability of Shekar being in the committee?

S – {3 out of 5 persons is to be chosen for a committee}

n(S) = $5C_3$

$^{n}C_r \ = \frac{n!}{(n-r)!r!}$

$^{5}C_3 \ = \frac{5!}{(5-3)!3!}$

$^{4}C_3 \ = \frac{5!}{2!3!} = \frac{5x4}{2}$

n(S) = 10

E – {Shekar being in the committee}

n (E) = $^{4}C_2$ x $1C_1$

Remaining	Shekar
$4C_2$	$1C_1$
6	1

n (E) = $\frac{4!}{(4-2)!2!}$x1

n (E) = $\frac{4!}{2!2!}$x1

n (E) = $\frac{4x3}{2}$x1

n (E) = 6

P (E) = $\frac{n(E)}{n(S)}$ ⇒ P (E) = $\frac{6}{10}$

3. Three cards are drawn at random from a pack of 52 cards. What is the probability that all the three cards are kings?

S – {Three cards are drawn at random from a pack of 52 cards}

n(S) = $^{52}C_3$

$^{n}C_r \ = \frac{n!}{(n-r)!r!}$

$^{52}C_3 \ = \frac{52!}{(52-3)!3!}$

$^{52}C_3 \ = \frac{52!}{49!3!}$

$^{52}C_3 \ = \frac{52x51x50}{6}$

$^{52}C_3 \ = \frac{52x51x50}{6}$

$^{52}C_3 \ = 22{,}100$

n(S) = 4

E – {All the three cards are kings}

n (E) = $4C_3$

n (E) = 4

P (E) = $\frac{n(E)}{n(S)}$

P (E) = $\frac{4}{22100}$

4. A committee of five persons is selected from 4 men and 3 women. What is the probability that the committee will have

(i) One man (ii) two men (iii) two women (iv) at least two men?

Sol: S – {a committee of five persons is selected from 4 men and 3 women}

n(S) = $^{7}C_5$

$nC_r \ = \frac{n!}{(n-r)!r!}$

$7C_5 = \frac{7!}{(7-5)!5!} = \frac{7!}{2!5!} = \frac{7x6}{2} = 21$

n(S) = 21

(i) E – {committee will have one man }

M	F
$4C_1$	$3C_4$

n (E) = 0

1	Not possible

$P(E) = \frac{n(E)}{n(S)} \Rightarrow P(E) = 0$

(ii)E – {Committee will have two men}

$n(E) = {}^4C_2 x {}^3C_3$

$n(E) = \frac{4!}{(4-2)!2!} x1 \Rightarrow n(E) = \frac{4!}{2!2!} x1 = 6x1 = 6$

$P(E) = \frac{n(E)}{n(S)} \Rightarrow P(E) = \frac{6}{21}$

M	F
$4C_2$	$3C_3$
6	1

(iii)E – {Committee will have two women}

$n(E) = {}^4C_3 x {}^3C_2 \Rightarrow n(E) = 4x3 \Rightarrow n(E) = 12$

$P(E) = \frac{n(E)}{n(S)} \Rightarrow P(E) = \frac{12}{21}$

M	F
$4C_3$	$3C_2$
4	3

(iv)E – {Committee will have at least two men}

$n(E) = {}^4C_2 x3C_3 + {}^4C_3 x3C_2 + {}^4C_4 x {}^3C_1$

$n(E) = 6x1 + 4x3 + 1x3$

$n(E) = 6 + 12 + 3 = 21$

$P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{21}{21} \Rightarrow P(E) = 1$

M	F	+	M	F	+	M	F
$4C_2$	$3C_3$		$4C_3$	$3C_2$		$4C_4$	$3C_1$
6	1		4	3		1	3